Warm-up!

1) Let f be the line passing through $(3,5)$ and $(9,2)$. Find $f(13)$.

2) Let h be a function passing through $(1,14)$ and $(5,23)$. Pretend h is a line, then find $h(8)$.

3) If $f(x)$ is an even function and $f(4) = -7$, find $f(-4)$.

3) If $g(x)$ is an odd function and $g(3) = -10$, find $g(-3)$.

Unit 1 Day 5 (1.5)

- Even and Odd Functions
- Systems of Equations

Homework

- Even/Odd Functions + Systems of Equations Worksheet

First Quiz of The Year: Wednesday/Thursday (the class after next). It includes everything except inequalities.

Even and Odd Functions

Even Functions

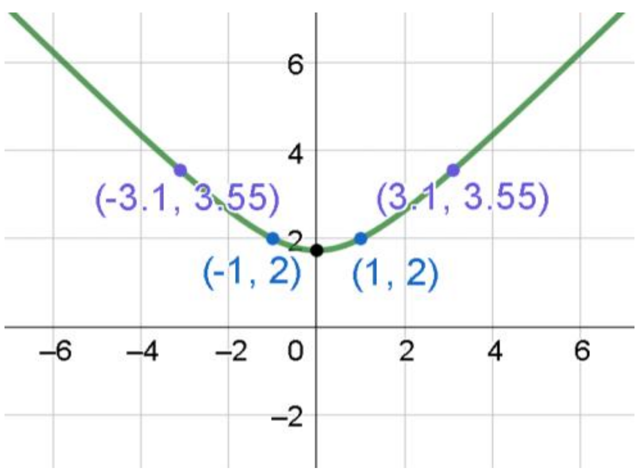
$$f(x) = \sqrt{x^2 + 3}$$

Numerically: Plugging in opposite x-values gives identical y-values

Graphically: y-axis symmetry

Function Composition: $f(-x) = f(x)$

Points: If (a, b) is on the graph, then $(-a, b)$ is also on the graph



Odd Functions

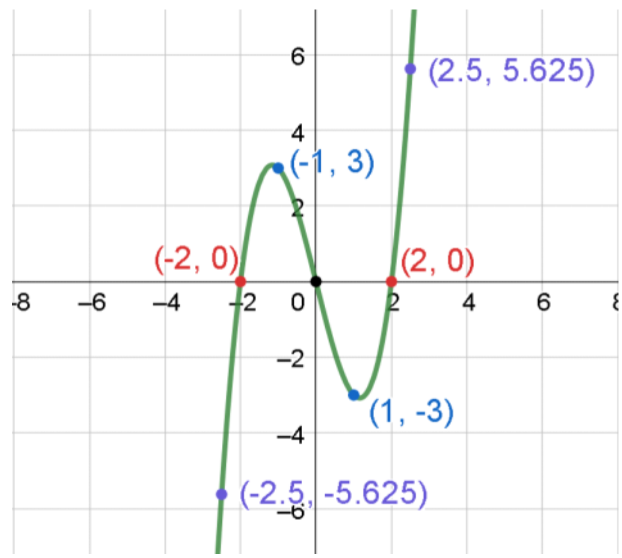
$$f(x) = x^3 - 4x$$

Numerically: Plugging in opposite x-values gives opposite y-values

Graphically: origin symmetry

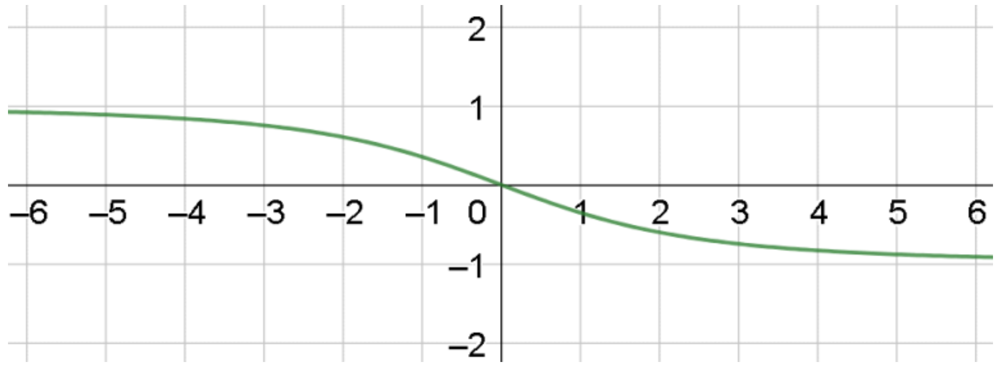
Function Composition: $f(-x) = -f(x)$

Points: If (a, b) is on the graph, then $(-a, -b)$ is also on the graph



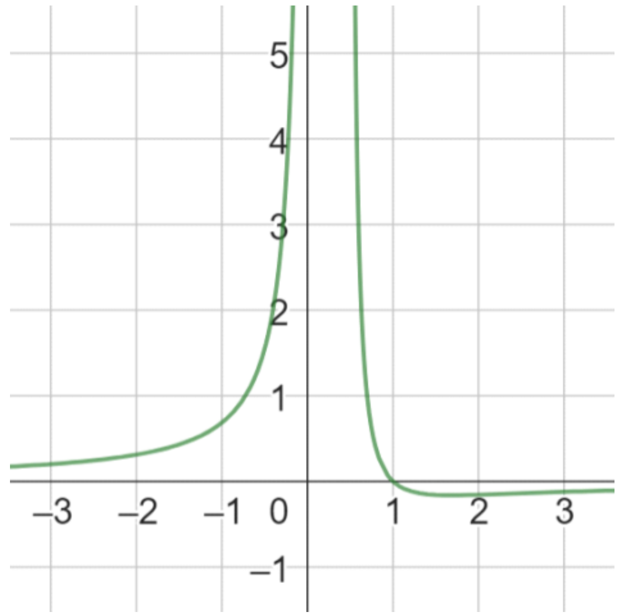
Determine if the function is even, odd, or neither

A) $h(x) = \frac{-x}{\sqrt{7+x^2}}$



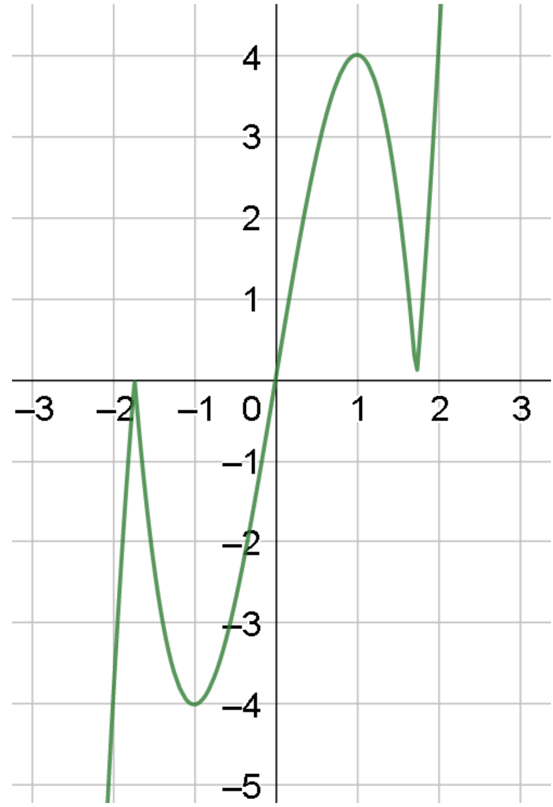
Determine if the function is even, odd, or neither

B) $f(x) = \frac{x-1}{x-2x^2}$



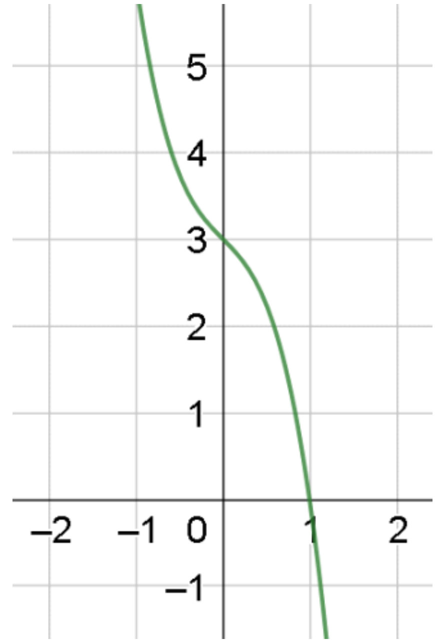
Determine if the function is even, odd, or neither

C) $g(x) = 2x\left|x^2 - 3\right|$



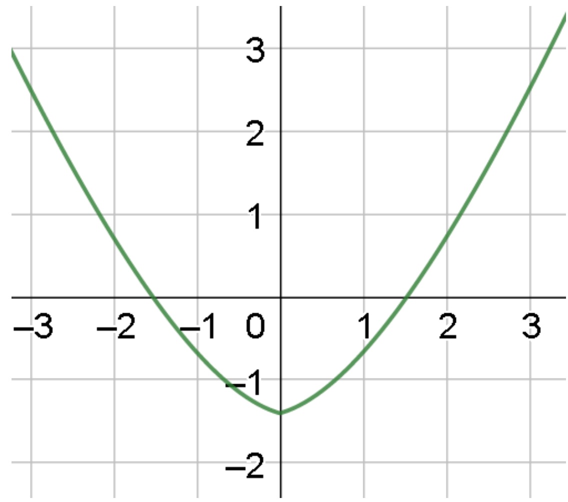
Determine if the function is even, odd, or neither

D) $f(x) = -2x^3 - x + 3$



Determine if the function is even, odd, or neither

E) $r(x) = \frac{3x^2-7}{|x|+5}$



Consequences of Being Even/Odd

Let $f(x)$ be an even function. What else do we know if…

A) $(-2,-3)$ is on the graph of $y = f(x)$

B) $f(x)$ is increasing on the interval $(2,5)$

C) $f(7) = -1$ is a relative minimum

D) $\lim_{x\to\infty} f(x) = -\infty$

E) The rate of change of $f(x)$ is increasing on the interval $(-8,-6)$

Consequences of Being Even/Odd

Let $g(x)$ be an odd function. What else do we know if…

F) The graph of g is tangent to the x-axis at $x = 5$

G) On the interval $(1,4)$, $g(x)$ is decreasing and concave down

H) $g(-7) = -5$ is a relative minimum

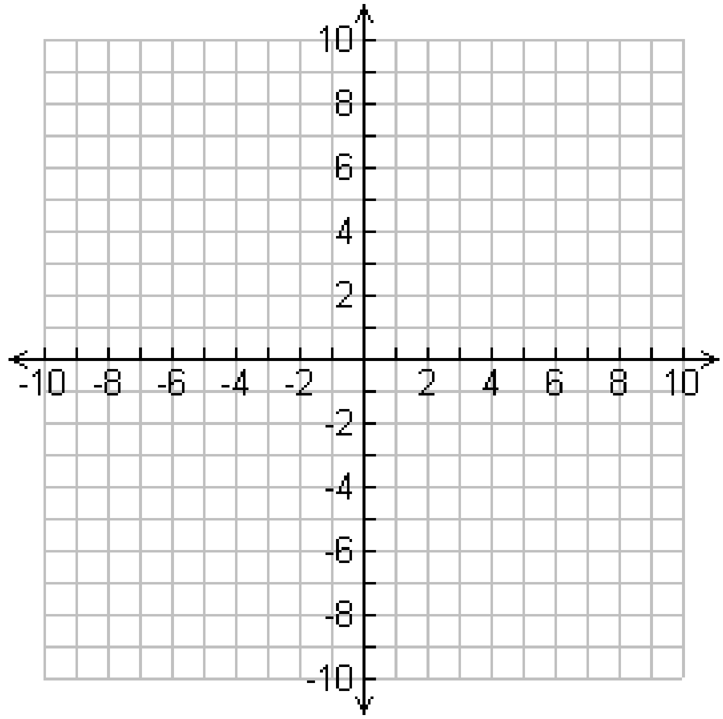
I) $\lim_{x \to -\infty} g(x) = -\infty$

Sketch a polynomial $f(x)$ with the given characteristics

- $f(x)$ is even
- x is a factor of $f(x)$
- $\lim_{x \to \infty} f(x) = -\infty$
- $f(-3) = 2$ is a local maximum

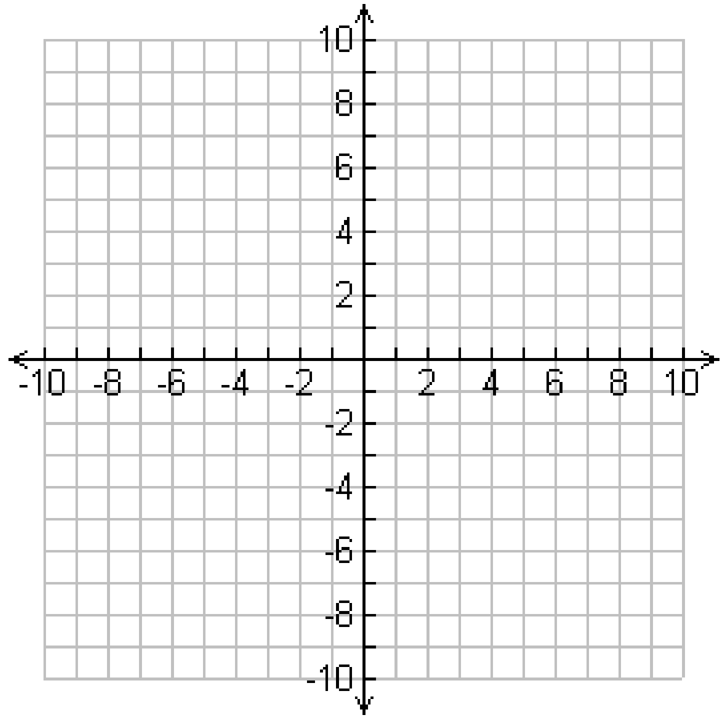
What is the minimum possible degree of this polynomial?

Is it possible this is a 6th degree polynomial? A 7th degree polynomial?



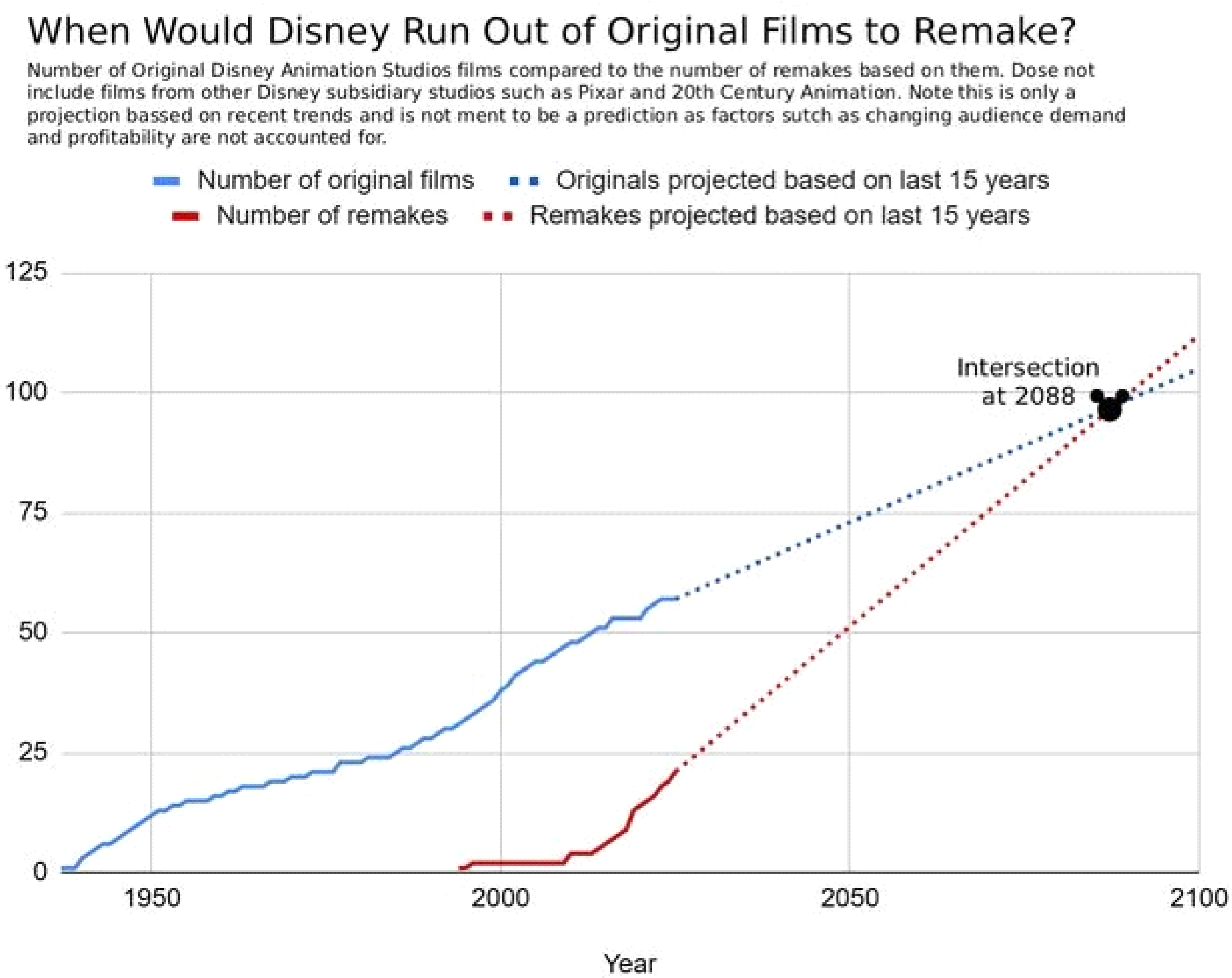
Sketch a polynomial $f(x)$ with the given characteristics

- $f(x)$ is odd
- $f(x)$ is cubic
- $2i$ is a zero of $f(x)$
- $\lim_{x \to -\infty} f(x) = -\infty$



Could $(0,8)$ be the y-intercept of this graph?

Linear Approximation



Three Ways to Perform Linear Approximation

Slope Formula

Consider a function $M(t)$ for which $M(2) = 8$ and $M(6) = 10$.

Find the average rate of change of M from $t = 2$ to $t = 6$.

Use this average rate of change to predict $M(12)$

Use this average rate of change to predict $M(-5)$

Consider the function $R(t) = -t^2 - 7t + 5$

Find $R(-1)$ and $R(2)$

Find the average rate of change of R over the interval $-1 \leq t \leq 2$

Apply that average rate of change to estimate $R(1.6)$.

Without actually finding $R(1.6)$, can we predict whether our estimate is an overapproximation or underapproximation?

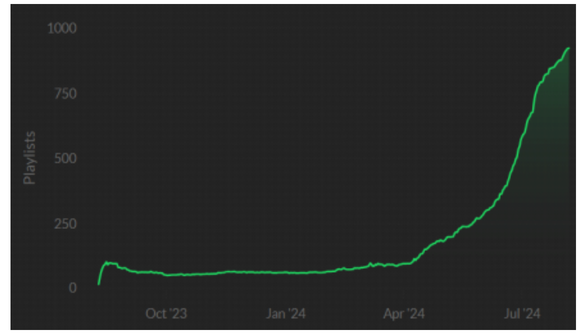
Let $h(x) = 2x^2 + x - 5$

(i) Find the average rate of change of h over the interval $-2 \leq x \leq 4$

(ii) Use the average rate of change found in (i) to estimate $h(5.2)$

(iii) Let $L(x)$ be a linear function that estimates the value of $h(x)$ based on the average rate of change computer in (ii). For $-2 \leq x \leq 4$, which is greater, the actual value of $h(x)$ or the linear approximation? Use concavity to explain how you know.

Chappell Roan's "HOT TO GO!" was released August 11, 2023. Three months after being released ($t = 3$), it could be found on 58 spotify playlists. Ten months after being released ($t = 10$), it could be found on 338 Spotify playlists. Let $N(t)$ be a function that includes both of the given data points to model how many Spotify playlists included "HOT TO GO!" t months after the song was released.



(i) Use the given data to find the average rate of change of $N(t)$, in playlists per month, from $t = 3$ to $t = 10$. Express your answer as a decimal approximation.

(ii) Use the average rate of change found in (i) to estimate the number of Spotify playlists that contained "HOT TO GO!" twelve months after the song's release.